STRUCTURE-PRESERVING DISCRETIZATIONS TO STOCHASTIC DIFFERENTIAL EQUATIONS

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In this talk, we focus our attention on structure-preserving numerical discretizations of selected stochastic differential equations (SDEs). The first part of this talk is devoted to the analysis of stochastic Runge-Kutta and ϑ -methods applied to mean-square dissipative SDEs [6], showing the mean-square contractive character of such discretizations, under suitable stepsize restrictions or algebraic constraints on the coefficients of the method [4, 5]. The second part of this talk investigates stochastic Hamiltonian systems (both of Itô and Stratonovich type [1, 2, 7]) under time discretizations. In particular, we present long-term estimates for the numerical Hamiltonian, highlighting the eventual conservative nature of the underlying numerical method [3]. Selected numerical results are provided to confirm the effectiveness of the theoretical analysis.

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